# A-LEVEL
# MATHEMATICS

Mechanics 5 – MM05
Mark scheme

6360
June 2014

Version/Stage: 1.0 Final

Mark schemes are prepared by the Lead Assessment Writer and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation events which all associates participate in and is the scheme which was used by them in this examination. The standardisation process ensures that the mark scheme covers the students' responses to questions and that every associate understands and applies it in the same correct way. As preparation for standardisation each associate analyses a number of students' scripts: alternative answers not already covered by the mark scheme are discussed and legislated for. If, after the standardisation process, associates encounter unusual answers which have not been raised they are required to refer these to the Lead Assessment Writer.

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**Key to mark scheme abbreviations**

| | |
|---|---|
| M | mark is for method |
| m or dM | mark is dependent on one or more M marks and is for method |
| A | mark is dependent on M or m marks and is for accuracy |
| B | mark is independent of M or m marks and is for method and accuracy |
| E | mark is for explanation |
| √or ft or F | follow through from previous incorrect result |
| CAO | correct answer only |
| CSO | correct solution only |
| AWFW | anything which falls within |
| AWRT | anything which rounds to |
| ACF | any correct form |
| AG | answer given |
| SC | special case |
| OE | or equivalent |
| A2,1 | 2 or 1 (or 0) accuracy marks |
| –*x* EE | deduct *x* marks for each error |
| NMS | no method shown |
| PI | possibly implied |
| SCA | substantially correct approach |
| c | Candidate |
| sf | significant figure(s) |
| dp | decimal place(s) |

**No Method Shown**

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

**Otherwise we require evidence of a correct method for any marks to be awarded.**

| Q | Solution | Mark | Total | Comment |
|---|---|---|---|---|
| **1** | $a\omega = 1.3$ | B1 | | Award B1 for $a^2\omega^2 = 1.3^2$ OE. |
| | $1.2^2 = \omega^2(a^2 - 0.2^2)$ | M1A1 | | M1: Equation with correct terms, but may contain sign errors.<br>A1: Correct equation. |
| | $1.2^2 = \left(\frac{1.3}{a}\right)^2 (a^2 - 0.2^2)$ | | | dM1: Solving for $a$.<br>A1: Correct $a$.<br>A1: Correct $AB$ |
| | $1.44 = 1.69 - \frac{0.0676}{a^2}$ | | | |
| | $a^2(1.69 - 1.44) = 0.0676$ | | | Accept $\frac{26}{25}$ |
| | $a = \sqrt{\frac{0.0676}{(1.69 - 1.44)}} = 0.52$ | dM1A1 | | |
| | $AB = 2 \times 0.52 = 1.04$ m | A1 | 6 | |
| | **Total** | | **6** | |

| Q | Solution | Mark | Total | Comment |
|---|---|---|---|---|
| **2(a)** | $ml\frac{\mathrm{d}^2\theta}{\mathrm{d}t^2} = -mg\sin\theta$ | M1A1 | | M1: Equation of motion with correct terms.<br>A1: Equation with correct terms and signs. |
| | No air resistance / $\sin\theta \approx \theta$ | B1 | | B1: Correct assumption.<br>(Allow sphere is a particle.) |
| | $\frac{\mathrm{d}^2\theta}{\mathrm{d}t^2} = -\frac{mg\theta}{ml}$ | | | |
| | $\frac{\mathrm{d}^2\theta}{\mathrm{d}t^2} = -\frac{g}{l}\theta$ **AG** | A1 | 4 | A1: Correct simplification to obtain final answer. |
| **b(i)** | $\frac{\mathrm{d}^2\theta}{\mathrm{d}t^2} = -\frac{9.8}{0.392}\theta = -25\theta$ | B1 | | B1: Obtaining $25\theta$<br>M1: Expression for $\theta$ with two unknown constants. May include their values instead of 5.<br>A1: Correct value for $B$<br>A1: Correct value for $A$ and correct final answer. |
| | $\theta = A\sin(5t) + B\cos(5t)$ | M1 | | |
| | $t = 0, \theta = \frac{\pi}{10} \Rightarrow B = \frac{\pi}{10}$ | A1 | | |
| | $t = 0, \dot{\theta} = 0 \Rightarrow A = 0$ | A1 | | |
| | $\theta = \frac{\pi}{10}\cos(5t)$ | | 4 | |
| **b(ii)** | $\frac{\pi}{15} = \frac{\pi}{10}\cos(5t) \Rightarrow \mathrm{t} = 0.16821$ | | | M1: Forming two equations using $\frac{\pi}{15}$ and $\frac{\pi}{30}$<br>A1: Correct equations<br>dM1: Obtaining two solutions.<br>A1: Correct final answer. |
| | $\frac{\pi}{30} = \frac{\pi}{10}\cos(5\mathrm{t}) \Rightarrow \mathrm{t} = 0.24619$ | M1A1 | | |
| | $0.24619 - 0.16821 = 0.0780$ | dM1<br>A1 | | |
| | | | 4 | Accept 0.078 |
| | **Total** | | **12** | |

| Q | Solution | Mark | Total | Comment |
|---|---|---|---|---|
| **3(a)** | $r = 3$ | B1 | 1 | B1: Correct value. |
| **(b)** | $r\dot{\theta} = 5$ | B1 | | B1: Statement of $r\dot{\theta} = 5$ |
| | $r^2\dot{\theta} = \text{Constant} = r \times r\dot{\theta} = 3 \times 5 = 15$ | M1 | | M1: Use of $r^2\dot{\theta} = \text{Constant}$ |
| | $\dot{\theta} = \frac{15}{r^2} = \frac{15}{9}(1+\sin\theta)^2$ | dM1 | | dM1: Solving for $\dot{\theta}$ |
| | $= \frac{5}{3}(1+\sin\theta)^2$ | A1 | 4 | A1: Correct expression for $\dot{\theta}$ from correct working |
| | **AG** | | | |
| **c(i)** | $\dot{r} = -3(1+\sin\theta)^{-2}\cos\theta\dot{\theta}$ | M1 | | M1: Differentiating $r$ wrt $t$ |
| | $= -5\cos\theta$ | A1 | 2 | A1: Correct expression for $\dot{r}$ |
| **c(ii)** | $\ddot{r} = 5\sin\theta\dot{\theta} = \frac{75\sin\theta}{r^2}$ | M1A1 | | M1: Differentiating $\dot{r}$ wrt $t$<br>A1: Correct expression for $\ddot{r}$. |
| | $\ddot{r} - r\dot{\theta}^2 = \frac{75\sin\theta}{r^2} - r \times \frac{225}{r^4}$ | dM1 | | dM1: Applying $\ddot{r} - r\dot{\theta}^2$ |
| | $= \frac{75\sin\theta}{r^2} - \frac{225}{r^3}$ | | | A1: Correct final answer with correct value of $k$. |
| | $= \frac{1}{r^2}(75\sin\theta - 225\frac{(1+\sin\theta)}{3})$ | | | |
| | $= -\frac{75}{r^2}$ | | | |
| | $k = -75$ | A1 | 4 | |
| | or | | | |
| | $\ddot{r} = 5\sin\theta\dot{\theta} = \frac{75\sin\theta}{r^2}$ | (M1A1) | | |
| | $\ddot{r} - r\dot{\theta}^2 = \frac{25}{3}\sin\theta(1+\sin\theta)^2 - \frac{3}{(1+\sin\theta)}\left(\frac{25}{9}\right)(1+\sin\theta)^4$ | (M1) | | |
| | $= \frac{25}{3}\left(\frac{3}{r}-1\right)\left(\frac{3}{r}\right)^2 - \frac{75}{9}\left(\frac{3}{r}\right)^3$ | | | |
| | $= -\frac{75}{r^2}$ | | | |
| | $k = -75$ | (A1) | | |
| | **Total** | | **11** | |

| Q | Solution | Mark | Total | Comment |
|---|---|---|---|---|
| **4(a)** | $m\frac{d^2x}{dt^2} = mg - T$ | M1 | | M1: Use of Newton's second Law with weight and tension. |
| | $= mg - \frac{mg}{0.2}(x - 0.2 - 0.1\sin(4t))$ | M1A1 | | M1: Expression for the tension with three terms for the extension.<br>A1: Correct expression for tension. |
| | $= mg(1 - 5x + 1 + 0.5\sin(4t))$ | | | M1: Rearranged to the required format. |
| | $\frac{d^2x}{dt^2} = -5gx + 2g + 0.5g\sin(4t)$ | M1 | | A1: Correct result from correct working. |
| | $\frac{d^2x}{dt^2} + 49x = 19.6 + 4.9\sin(4t)$ **AG** | A1 | 5 | |
| **(b)** | **CF** | | | |
| | $x = D\sin(7t) + E\cos(7t)$ | B1 | | B1: Correct CF with two unknown constants. |
| | **PI** | | | |
| | $x = A + B\sin(4t) + C\cos(4t)$ | M1 | | M1: Correct form of PI |
| | $\dot{x} = 4B\cos(4t) - 4C\sin(4t)$ | | | dM1: Correct derivatives of PI |
| | $\ddot{x} = -16B\sin(4t) - 16C\cos(4t)$ | dM1 | | B1: Correct constant term. |
| | $49A + 33B\sin(4t) + 33C\sin(4t)$ | | | |
| | $= 19.6 + 4.9\sin(4t)$ | | | |
| | $A = \frac{19.6}{49} = 0.4$ | B1 | | |
| | $B = \frac{4.9}{33} = \frac{49}{330}$ | | | M1: Attempting to find $B$ and $C$. |
| | $C = 0$ | M1A1 | | A1: $B$ and $C$ correct. |
| | $x = 0.4 + \frac{49}{330}\sin(4t)$ | | | |
| | $x = D\sin(7t) + E\cos(7t) + 0.4 + \frac{49}{330}\sin(4t)$ | | | M1: Using initial position to find $E$.<br>A1: Correct $E$. |
| | $x = 0.4, t = 0$ | M1 | | |
| | $E = 0$ | A1 | | |
| | $v = 7D\cos(7t) + \frac{196}{330}\cos(4t)$ | M1 | | M1: Using initial velocity to find $D$. |
| | $v = 0, t = 0$ | | | A1: Correct $D$. |
| | $D = -\frac{28}{330}$ | | | |
| | $x = -\frac{28}{330}\sin(7t) + 0.4 + \frac{49}{330}\sin(4t)$ | A1 | 10 | |
| | **Total** | | **15** | |

| Q | Solution | Mark | Total | Comment |
|---|---|---|---|---|
| **5(a)** | $(m+\delta m)(v+\delta v)+(-\delta m)(v-U)-mv=-mg\delta t$<br>$mv+m\delta v+v\delta m+-v\delta m+U\delta m-mv=-mg\delta t$<br>$m\delta v+U\delta m=-mg\delta t$<br>$m\frac{\delta v}{\delta t}+U\frac{\delta m}{\delta t}=-mg$<br>$m\frac{dv}{dt}+U\frac{dm}{dt}=-mg$<br>$\frac{dv}{dt}=-\frac{U}{m}\frac{dm}{dt}-g$ **AG** | M1A1<br><br><br><br>dM1<br><br>A1 | 4 | M1: Correct terms, with possible sign errors.<br>A1: Correct equation.<br>dM1: Simplification.<br>A1: Correct result from correct working. |
| **(b)** | But : $m=M-\lambda t$ and $\frac{dm}{dt}=-\lambda$<br>$\frac{dv}{dt}=\frac{\lambda U}{M-\lambda t}-g$ **AG** | M1<br>A1 | 2 | M1: Use of expressions for $m$ and $\frac{dm}{dt}$ |
| **(c)** | $v=\int\left(\frac{\lambda U}{M-\lambda t}-g\right)dt$<br>$=-U\ln(M-\lambda t)-gt+c$<br>$v=0,t=0\Rightarrow c=U\ln(M)$<br>$v=U\ln(M)-U\ln(M-\lambda t)-gt$<br>$=U\ln\left(\frac{M}{M-\lambda t}\right)-gt$ | <br><br>M1A1<br><br>M1A1<br><br>A1 | 5 | M1: Integration to give ± correct ln term.<br>A1: Correct integration<br>M1: Finding constant of integration.<br>A1: Correct constant<br>A1: Correct expression |
| **(d)** | $M-\lambda t=\frac{M}{5}$<br>$t=\frac{4M}{5\lambda}$<br>$v=U\ln\left(\frac{M}{M-\frac{4M}{5}}\right)-\frac{4Mg}{5\lambda}$<br>$=U\ln 5-\frac{4Mg}{5\lambda}$ | M1<br><br>A1<br><br>M1<br><br>A1 | 4 | M1: Equation for time with $\frac{M}{5}$<br>A1: Correct time<br>M1: Finding $v$<br>A1: Correct $v$.<br>Accept $1.61U-0.8\frac{Mg}{\lambda}$ OE |
| | **Total** | | **15** | |

| Q | Solution | Mark | Total | Comment |
|---|---|---|---|---|
| **6. (a)** | $EPE = \frac{4mg}{2a}\left(2a\sin\left(\frac{\theta}{2}\right) - a\right)^2$ | M1A1 | | M1: Attempt at EPE<br>A1: Correct EPE |
| | $GPE = \frac{mga}{2}\cos\theta$ | B1 | | B1: Correct GPE. |
| | $V = \frac{mga}{2}\left(16\sin^2\left(\frac{\theta}{2}\right) - 16\sin\left(\frac{\theta}{2}\right) + 4 + \cos\theta\right)$<br>$= \frac{mga}{2}\left(8(1-\cos\theta) - 16\sin\left(\frac{\theta}{2}\right) + 4 + \cos\theta\right)$ | dM1 | | dM1: Finding total and simplifying.<br>A1: Correct final answer from correct working. |
| | $= \frac{mga}{2}\left(12 - 7\cos\theta - 16\sin\left(\frac{\theta}{2}\right)\right)$ **AG** | A1 | 5 | |
| **(b)** | String must be taut for expression to be valid. (As the natural length of the string is equal to the radius an equilateral triangle will be formed when the string is just taut and so the inequality must hold and is needed on both sides of the vertical.) | B1 | 1 | B1: Mentioning that the string must be taut. |
| **(c)** | $\frac{dV}{d\theta} = 0$ | | | M1: Differentiation.<br>A1: Correct derivative |
| | $0 = 7\sin\theta - 8\cos\left(\frac{\theta}{2}\right)$ | M1A1 dM1 | | dM1: Setting derivative equal to zero. |
| | $0 = 14\sin\left(\frac{\theta}{2}\right)\cos\left(\frac{\theta}{2}\right) - 8\cos\left(\frac{\theta}{2}\right)$ | | | |
| | $0 = \cos\left(\frac{\theta}{2}\right)\left(14\sin\left(\frac{\theta}{2}\right) - 8\right)$ | | | dM1: Solving for $\theta$. |
| | $\cos\left(\frac{\theta}{2}\right) = 0$ or $\sin\left(\frac{\theta}{2}\right) = \frac{4}{7}$ | dM1 | | A1: One correct solution.<br>A1: Two other correct solutions |
| | $\theta = \pi$ or 1.22 or 5.07 | A1A1 | 6 | |
| **(d)** | $\frac{d^2V}{d\theta^2} = \frac{mga}{2}\left(7\cos\theta + 4\sin\left(\frac{\theta}{2}\right)\right)$ | M1 | | M1: Correct second derivative. |
| | $\theta = 1.22$ $\frac{d^2V}{d\theta^2} = (+4.7...)\frac{mga}{2}$ $\therefore$ Stable | A1 | | A1: One correct explanation and conclusion. |
| | $\theta = \pi$ $\frac{d^2V}{d\theta^2} = (-3)\frac{mga}{2}$ $\therefore$ Unstable | B1 | | B1: Correct explanation and conclusion for $\theta = \pi$. |
| | $\theta = 5.07$ $\frac{d^2V}{d\theta^2} = (+4.7...)\frac{mga}{2}$ $\therefore$ Stable | A1 | 4 | A1: Correct explanation and conclusion for third solution. |
| | **Total** | | **16** | |
| | **TOTAL** | | **75** | |